Where Does the Volume Term Come From?

We want to look at attributes of animals independent of measured power

For single targets this means: TS or σ_{bs}

We need an equivalent measure for a collection of scatterers:

$$p_{scat}^{\ 2} = (p_0 r_0)^2 \left(\frac{1}{r_{target}}\right)^2 \left(\frac{1}{r_{source}}\right)^2 \sigma_{bs}$$

For n scatterers:

$$np_{scat}^{\ 2} = (p_0 r_0)^2 \left(\frac{1}{r_{target}}\right)^2 \left(\frac{1}{r_{source}}\right)^2 \sigma_{bs} n$$

Define number of scatterers per unit volume of water as:

$$N = \frac{n}{V}$$

$$n = NV$$

Substitute in:

$$np_{scat}^{\ 2} = (p_0 r_0)^2 \left(\frac{1}{r_{target}}\right)^2 \left(\frac{1}{r_{source}}\right)^2 N\sigma_{bs} V$$

Remembering definitions:

$$s_v = N\sigma_{bs}$$

$$S_v = 10\log_{10}(s_v)$$

Substituting in for volume backscattering coefficient:

$$np_{scat}^{\ 2} = (p_0 r_0)^2 \left(\frac{1}{r_{target}}\right)^2 \left(\frac{1}{r_{source}}\right)^2 s_v V$$

Logarithmic form of the sonar equation for volume backscatter:

$$\text{EL} = \text{SL} - 2\text{TL} + S_v + 10\log_{10}(V)$$

What are the transmission losses?

EL = SL - 2TL + S_v + 10 $\log_{10}$(V)

Start with linear pressure terms:

$$np_{scat}^{\ 2} = (p_0 r_0)^2 \left(\frac{1}{r_{target}}\right)^2 \left(\frac{1}{r_{source}}\right)^2 N\sigma_{bs} V$$

$$np_{scat}^{\ 2} = (p_0 r_0)^2 \left(\frac{1}{r^4}\right) N\sigma_{bs} V$$

By definition:

$$V = r^2 \left(\frac{\psi c \tau}{2}\right)$$

Substitute terms for volume:

$$np_{scat}^{\ 2} = (p_0 r_0)^2 \left(\frac{1}{r^4}\right)\left(r^2 \left(\frac{\psi c \tau}{2}\right) s_v\right)$$

Which reduces to:

$$np_{scat}^{\ 2} = (p_0 r_0)^2 \left(\frac{1}{r^2}\right)\left(\frac{\psi c \tau}{2}\right) s_v$$

Back to logs:

$$10\log_{10}\left(np_{scat}^{\ 2}\right) = 10\log_{10}\left((p_0 r_0)^2\right) + 10\log_{10}\left(\frac{1}{r^2}\right) + 10\log_{10}\left(\frac{\psi c \tau}{2}\right) + 10\log_{10}(s_v)$$

Finally:

$$10\log_{10}\left(np_{scat}^{\ 2}\right) = 10\log_{10}\left((p_0 r_0)^2\right) - 20\log_{10}(r) + 10\log_{10}(V) + 10\log_{10}(s_v)$$